AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Question Set 3

1 Solve the equation $4x^{-\frac{1}{2}} = 7$, giving your answer as a fraction in its lowest terms. [3]

$$\frac{4}{\sqrt{x}} = 7 \rightarrow \frac{4}{7} = \sqrt{x} \rightarrow \frac{16}{49} = x.$$

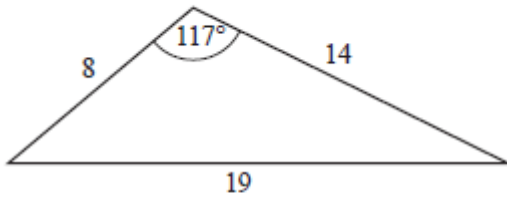**2** Fig. 2 shows a triangle with one angle of 117° given. The lengths are given in centimetres.

Fig. 2 $\frac{1}{2}AB\sin C$
$= \frac{1}{2} \times 14 \times 8 \times \sin 117 = 49.9 cm^2$

Calculate the area of the triangle, giving your answer correct to 3 significant figures. [2]

3 Without using a calculator, prove that $3\sqrt{2} > 2\sqrt{3}$. [3]

$3\sqrt{2} = \sqrt{9 \times 2} = \sqrt{18}$ $\sqrt{18} > \sqrt{12}$

$2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$ $\therefore 3\sqrt{2} > 2\sqrt{3}$

4 The equation of a circle is $x^2 + y^2 + 8x - 6y - 39 = 0$.

$(x+4)^2 + (y-3)^2 = 64$

(a) Find the coordinates of the centre of the circle. $(-4,3)$ [2]

(b) Find the radius of the circle. 8 [1]

$\sqrt{64} = 8$

5 (a) Find $\int x^3 \left(15x + \frac{11}{3\sqrt{x}}\right) dx$. $\int 15x^4 + 11x^{\frac{8}{3}} dx = 3x^5 + 3x^{\frac{11}{3}} + C$ [5]

(b) Show that $\int_0^8 x^3 \left(15x + \frac{11}{3\sqrt{x}}\right) dx = a \times 2^{11}$, where a is a positive integer to be determined. [3]

$\left[3x^5 + 3x^{\frac{11}{3}}\right]_0^8$ $\frac{104448}{2^{11}} = 51$

$(98304 + 6144) - 0$ $a = 51$

$= 104448$ $= 51 \times 2^{11}$

6

In 2012 Adam bought a second hand car for £8500. Each year Adam has his car valued. He believes that there is a non-linear relationship between t, the time in years since he bought the car, and V, the value of the car in pounds. Fig. 6.1 shows successive values of V and $\log_{10}V$.

t	0	1	2	3	4
V	8500	6970	5720	4690	3840
$\log_{10}V$	3.93	3.84	3.76	3.67	3.58

Fig. 6.1

Adam uses a spreadsheet to plot the points $(t, \log_{10}V)$ shown in Fig. 6.1, and then generates a line of best fit for these points. The line passes through the points $(0, 3.93)$ and $(4, 3.58)$. A copy of his graph is shown in Fig. 6.2.
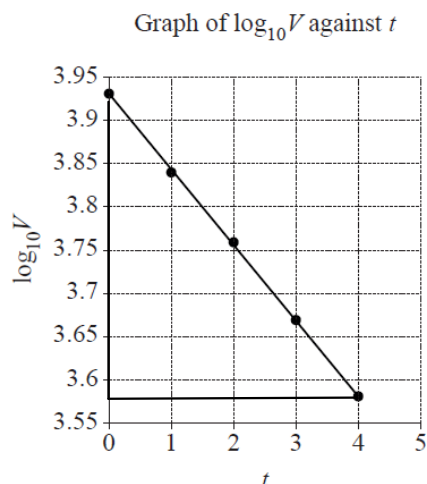
Graph of $\log_{10}V$ against t

Fig. 6.2

(a) Find an expression for $\log_{10}V$ in terms of t. [3]

Gradient $= \dfrac{3.58 - 3.93}{4 - 0} = -\dfrac{7}{80}$

when $t = 0$, $\log V = 3.93$ (y-intercept)

$\log V = -\dfrac{7}{80}t + 3.93$

(b) Find a model for V in the form $V = A \times b^t$, where A and b are constants to be determined. Give the values of A and b correct to 2 significant figures. [3]

$\log A = 3.93$, $\therefore A = 8511 = 8500 \ (2sf)$

$\log B = -\dfrac{7}{80}$, $\therefore B = 0.818 = 0.82$ (2sf)

$\therefore V = 8500 \times 0.82^t$

In 2017 Adam's car was valued at £3150.

(c) Determine whether the model is a good fit for this data. [1]

at $t = 5$, $V = 3108$] $3108 \neq 3150$ $\therefore$ not a good fit

A company called Webuyoldcars pays £500 for any second hand car. Adam decides that he will sell his car to this company when the annual valuation of his car is less than £500.

(d) According to the model, after how many years will Adam sell his car to Webuyoldcars? [3]

$8511 \times 0.818^t < 500$

$0.818^t < 0.0587$

$t \log 0.818 < \log 0.0587$ $t = 15$

$t > 14.1$ $\therefore$ $t = 15$

$\therefore$ it will take
Adam 15 years

7

In this question you must show detailed reasoning.

The equation of a curve is $y = \frac{x^2}{4} + \frac{2}{x} + 1$. A tangent and a normal to the curve are drawn at the point where $x = 2$. $\leftarrow y = 3$

Calculate the area bounded by the tangent, the normal and the x-axis. [10]

① at the point $x = 2$

$y = \frac{4}{4} + \frac{2}{2} + 1 = 3$

② $\frac{dy}{dx} = \frac{x}{2} - \frac{2}{x^2}$, when $x = 2$, $\frac{dy}{dx} = \frac{2}{2} - \frac{2}{2^2} = 1 - \frac{1}{2} = \frac{1}{2}$

③ gradient of tangent $= \frac{1}{2}$

gradient of normal $= -2$

↓

Equation of the line.

$y = -2x + c$

⟹ $3 = -2(2) + c$

$3 = -4 + c$

$c = 7$

∴ $y = -2x + 7$

④ when $y = 0$, $x = ?$

$0 = -2x + 7$

$2x = 7$

$x = \frac{7}{2}$

ENLARGED DIAGRAM

(2, 3)

R

(2, 0)   ($\frac{7}{2}$, 0)
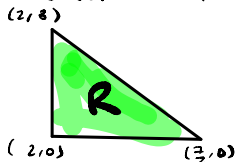
area $= \frac{1}{2}\left(\frac{7}{2} - 2\right)(3)$

$= \frac{9}{4} = 2.25$

Total Marks for Question Set 3: 39 marks